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INTERNAL BALLISTICS OF POWDER DRIVEN ROCKETS

by

Emory Lakatos

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NATIONAL DEFENSE RESEARCH COMMITTEE

REPORT NO. A-22 : PROGRESS REPORT

INTERNAL BALLISTICS OF POWDER DRIVEN ROCKETS

by

Emory Lakatos

Approved December 16, 1941
for submission to the Section Chairman

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Preface

This report is pertinent to the project designated by the War Department Liaison Officer as OD-26. It pertains to work in progress at Indian Head, Maryland, as a joint project of the Bureau of Ordnance, U.S. Navy, and Section H, Division A, National Research Committee.

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INTERNAL BALLISTICS OF POWDER DRIVEN ROCKETS[1/]

Abstract

An approximate theory of the internal ballistics of rockets driven by colloidal propellants is developed in this report. A number of approximate relationships are derived which may be of use in preliminary design work. Particular emphasis is placed on the case of a constant chamber pressure. Included in the report are sample design calculations for a 3-in. 15-lb shell. The calculations are summarized on graphs which show the weight of TNT that can be carried as a function of the propellant charge weight, chamber pressure and muzzle velocity. The results show that the largest weight of TNT can be carried when the operating pressure is between 25 and 50 atmos. This result is then shown to apply equally well to all shells geometrically similar to the one considered.

The theory developed in this report is necessarily approximate in the following respects:

(a) In formulating the law of burning, heat losses in walls are neglected. This means that no account is taken of the fact that the mean temperature of the gas in the rocket chamber is less than the temperature of the layer of gas immediately adjacent to the burning surface.

(b) The mass of gas discharged through the nozzle of the rocket is calculated on the assumption that the steady flow formulas may be applied to variable flow problems on a quasi-steady state basis. A more rigorous treatment of this problem does not seem to be available at this time.

1/ This report is a revision of one of October 29, 1940. For other material pertinent to the present subject, see C. N. Hickman, NDRC Report A-4, Appendix D; C. N. Hickman, Memos. A-18M to A-22M; J.W.M. DuMond, NDRC Report A-24. The last-named report is a relatively elementary treatment of the mechanical efficiency of rockets.

(c) The variation in gas temperature in the initial stages of burning is ignored. While this simplification yields a correct prediction of the equilibrium pressure, it may at times lead to incorrect results in the prediction of how soon equilibrium is reached.

(d) Effects of friction are ignored.

The English system of units based on the pound as the unit of force and slug as the unit of mass is used throughout the report. The list of symbols used is given in the Appendix.

1. Basic equation

The basic equation connecting the rate of burning and the rate of gas discharge is that the mass burned up to time *t* is equal to the mass of gas in the chamber plus the total mass of gas discharged. An equivalent statement is that the rate at which the propellant is consumed equals the rate at which the gas is discharged plus the rate at which the mass of gas inside the chamber changes; in symbols,

$$\dot{m} = \dot{m}_d + (dm_c/dt), \qquad (1)$$

where $\dot{m}$ is the mass (slugs) of propellant burned per second, $\dot{m}_d$ is the mass (slugs) of gas discharged per second and m_c is the mass (slugs) of gas in the chamber at the time *t*.

2. Law of burning

From the outset we shall restrict the treatment to the case of constant burning surface, such as a tube burning on its inside and outside curved surfaces, or a solid cylinder burning

only on its flat ends. Let D be the dimension of the propellant along the line of burning. This quantity is so defined that $\frac{1}{2}D$ is the greatest depth burned away below any receding surface before the whole grain is consumed. In the case of a tube, D is the wall thickness. For a cylinder burning from both of its flat ends, D is the length of the cylinder. If the cylinder burns from only one end, D is taken as twice the length of the cylinder. It is easy to see that, in the cases cited, the definition is satisfied. For example, in the tube, the wall burns a distance $\frac{1}{2}D$ from the inside and a distance $\frac{1}{2}D$ from the outside.

With this understanding, we take as the law of burning,

$$D\,\frac{dZ}{dt} = b + BP, \qquad (2)$$

where Z is the fraction of the propellant consumed at time t, b and B are constants of the specific propellant used, and P is the chamber pressure (lb/ft^2). The physical meaning of Eq. (2) can be shown as follows.

If it is assumed that the burning takes place by parallel layers and that the total burning surface is constant, the fractional mass of propellant consumed will be proportional to $D \cdot Z$, the total depth burned away. Hence $D(dZ/dt)$ must be the total distance that the flame travels per second. That is to say, if the flame velocity is, say, 1 ft/sec, then $D(dZ/dt)$ will be 2 ft/sec for the two burning surfaces. It is also clear that b has the dimensions of a velocity (ft/sec) and that B is a velocity per unit of pressure [(ft/sec)/(lb/ft^2)].

It should be remarked that this form of the burning law has been strongly criticized in the past because it is not correct either physically or philosophically.[2] However, the writer has found that it appears to be the only simple relationship using the burning constants determined from closed-bomb experiments that correctly predicts the equilibrium pressure in rockets. For example, the theoretical law derived by Crow and Grimshaw[2] fits the experiments with bombs but for rockets gives materially incorrect values of the equilibrium pressure attained as a function of the ratio of burning area to throat area. Just why apparently equivalent laws should give different results is not yet clear to the writer.

3. <u>Additional relationships</u>

The remaining equations necessary for the formulation of the problem are

$$\dot{m} = m_\tau dZ/dt, \qquad (3)$$

where m_τ is the initial mass of the propellant. The gas equation, assuming that the gas can be regarded as an ideal gas, is

$$Pv = RT, \qquad (4)$$

where P is the pressure (lb/ft^2), v is the specific volume (ft^3/slug), R is the gas constant (R = 1930 ft lb/slug $^\circ$F) and T is the combustion temperature ($^\circ$F absolute).

2/ In particular, see Crow and Grimshaw, Phil. Trans. Roy. Soc. (London) <u>230</u>, A691, pages 387-411 (1932).

Also, the instantaneous value of the mass of gas in the chamber is given by

$$m_c = V/v, \qquad (5)$$

where V (ft^3), the instantaneous gas volume in the chamber, may be expressed as

$$V = V_o + (m_t/\delta)\, Z = V_o + V_p \cdot Z, \qquad (6)$$

V_o being the initial, or clearance, volume (ft^3) V_p, the volume of the propellant (ft^3) and δ the density of the propellant ($slug/ft^3$).

Finally, the rate of gas discharge may be written as

$$\dot{m}_d = kP, \qquad (7)$$

if the chamber pressure does not fall below about 4300 lb/ft^2, or 2 atmos, above zero pressure. The constant k has the value

$$k = A_t \sqrt{\frac{\gamma}{RT} \left[\frac{2}{\gamma + 1} \right]^{(\gamma + 1)/(\gamma - 1)}} \qquad (8)$$

or

$$k = \beta A_t, \qquad (8a)$$

where A_t is the throat area of the nozzle (ft^2), γ is the adiabatic constant -- that is, the ratio of the specific heats at constant pressure and volume -- and the factor β, which represents the quantities under the radical in Eq. (8), is a constant that depends on the properties of the specific propellant used.

4. Equilibrium pressures

By combining Eqs. (4), (5) and (6) we can write the expression for the mass of gas in the chamber as

$$m_c = (V_o + V_p Z)P/RT. \qquad (9)$$

Equation (1) may then be written as

$$\dot{m} = \dot{m}_d + \frac{1}{RT} \frac{d}{dt} [(V_o+V_pZ)P]$$

$$= \dot{m}_d + \frac{1}{RT} \frac{d}{dZ} [(V_o+V_pZ)P] \frac{dZ}{dt}$$

or, upon eliminating dZ/dt by means of Eq. (3),

$$\dot{m} = \dot{m}_d + (1/RT)\left[P\, V_p + (V_o+V_pZ) \frac{dP}{dZ} \right] (\dot{m}/m_\tau).$$

Solving for the derivative dP/dZ, we obtain

$$\frac{dP}{dZ} = \frac{RTm_\tau[1 - (\dot{m}_d/\dot{m})] - P\, V_p}{V_o + V_p\, Z}. \tag{10}$$

For equilibrium, dP/dZ must be zero. The equilibrium pressure is therefore given by

$$P_e = \frac{RTm_\tau}{V_p} \left(1 - \frac{\dot{m}_d}{\dot{m}}\right).$$

But m_τ/V_p is the density δ of the propellant, and $RT\delta$ is the bomb pressure that would exist with 100 percent density of loading if the temperature were that for burning at constant pressure. Let this pressure be denoted by P_b. Then

$$P_e = P_b \left(1 - \frac{\dot{m}_d}{\dot{m}}\right). \tag{11}$$

This result is important and requires some comment. First of all, it holds independently of any particular law of burning. It is true whether the temperature transient is taken into account or not. It shows that, for an equilibrium pressure to exist, the rate of discharge must be less than the rate of burning. Finally, it shows that the value of the equilibrium pressure is apt to be critical as to factors that affect either

the burning or the discharge rates. This is so because P_b is of the order of 200,000 lb/in.^{2}, and the value of P_e desired for rocket work hardly ever reaches 10,000 lb/in.^{2}

If the burning law, Eq. (2), and discharge rate law, Eq. (7), are substituted into Eq. (10), there results

$$1 - \frac{k\,P_e}{\frac{m_\tau B}{D}\left(\frac{b}{B} + P_e\right)} - \frac{P_e}{P_b} = 0.$$

Let

$$m_\tau B/D = C, \qquad (12)$$

$$b/BP_b = \alpha, \qquad (13)$$

and

$$P_e/P_b = \lambda_e.$$

The equation that gives the (dimensionless) equilibrium pressure ratio explicitly is

$$\lambda_e^2 + \lambda_e[(k/C) + \alpha - 1] - \alpha = 0. \qquad (14)$$

Instead of solving for λ_e, it is more convenient to solve for $\underline{k/C}$ in terms of the equilibrium value; thus

$$k/C = (\alpha/\lambda_e) + 1 - \alpha - \lambda_e. \qquad (15)$$

If $\underline{S}$ is the total burning area,

$$m_\tau = \tfrac{1}{2}\,\delta\,SD. \qquad (15a)$$

Then

$$k/C = 2\beta A_t/\delta SB,$$

or

$$\frac{A_t}{S} = \frac{\delta B}{2\beta}\left(\frac{\alpha}{\lambda_e} + 1 - \alpha - \lambda_e\right). \qquad (16)$$

5. Variation of pressure with S/A_t, the ratio of burning area to throat area

To see how these considerations work out, consider a propellant particularly well adapted for rocket work; namely, one having a composition of 40 percent nitroglycerine and 60 percent nitrocellulose, without any stabilizer, and having properties approximately as follows:

$\underline{\gamma} = 1.25$,

$\underline{R} = 1930$ lb ft/slug $^{\circ}$F,

$\underline{T} = 5050^{\circ}$F absolute, at constant pressure,

$\underline{b} = 0.065$ ft/sec,

$\underline{B} = 4.5 \times 10^{-7}$ (ft/sec)/(lb/ft^2),

$\underline{\delta} = 3.10$ slug/ft^3 [= 100 lb of mass/ft^3].

Then, from Eq. (4), $P_b = 3.02 \times 10^7$ lb/ft^2, or 210,000 lb/in.^{2}; from Eqs. (8) and 8(a), $\beta = 11.97 \times 10^{-4}$ slug/lb sec; from Eq. (13), $\alpha = 4.80 \times 10^{-3}$.

By assuming successive values of λ_e, that is, of P_e/P_b, and substituting in Eq. (16), we get the curve in Fig. 4 showing the equilibrium chamber pressure in terms of the ratio S/A_t. The circles and squares show experimental points for two batches of powder. The agreement with the theoretical curve is quite good. It will be noted that, at low pressures, the rate of change of pressure with area ratio S/A_t is small. However, at the higher pressures, the actual pressure is quite sensitive to comparatively small changes in the area ratio.

6. Variation of pressure with fraction of propellant consumed

Returning now to the general equation [Eq. (10)] we can rewrite it in the dimensionless form

$$\frac{dZ}{(V_o/V_p) + Z} = - \frac{d\lambda}{\lambda^2 + \lambda[(k/C) + \alpha - 1] - \alpha} \qquad (17)$$

Let us assume that the pressure is initially brought up to some value P_o by means of a black powder charge. Then Eq. (17) can be integrated if we note that

$$\lambda = \lambda_o = P_o/P_b, \text{ when } Z = 0.$$

The solution is

$$\frac{\lambda}{\lambda_e} = 1 + \frac{[(\lambda_o/\lambda_e)-1]\,(\lambda_e^2 + \alpha)}{(\lambda_o\lambda_e + \alpha)\left(1 + \frac{V_p}{V_o}Z\right)^{(\lambda_e^2+\alpha)/\lambda_e} - (\lambda_o\lambda_e - \lambda_e^2)} .$$

This rather formidable looking equation purports to show how the actual pressure approaches the equilibrium pressure as a function of the fraction burned. It will be noticed that if $\lambda_o = \lambda_e$ -- that is, if the black powder charge starts the combustion off at just the right pressure -- the pressure throughout the burning interval will be the equilibrium pressure. For other values of the initial pressure, the results are more complex. Figure 3 shows λ/λ_e plotted as a function of $(V_p/V_o)Z$ for pairs of values of λ_e and λ_o/λ_e.

It will be noted that the curves of Fig. 3 have a dual significance. The abscissa is $(V_p/V_o)Z$. Hence for $Z = 1$ the curves give the pressures attained for various densities of loading at the end of the combustion period. For $Z \ll 1$ and

constant loading density the curves show the manner in which the pressure builds up with the fractional mass of propellant consumed. For example, consider the curve marked $\lambda_e = 0.01$, $\lambda_o/\lambda_e = 0.5$. This means that we are considering the case where the equilibrium pressure is $0.01 \times 210{,}000$, or 2100 lb/in.^{2} Suppose the design is such that $V_p/V_o = 2$, which corresponds to a density of loading of 66.7 percent. Then, since the propellant is all consumed when $Z = 1$, we see that, for $(V_p/V_o)Z = 2$, $\lambda/\lambda_e = 0.7$, or the pressure reached is 1470 lb/in.^{2}; that is, the equilibrium pressure is not reached (according to this equation) during the combustion process. If we wish to know the pressure when the combustion is 30 percent complete, $Z = 0.3$, and $(V_p/V_o)Z = 0.6$. The ordinate is 0.605 and hence the pressure reached is $0.605 \times 210{,}000$, or 1270 lb/in.^{2} If the density of loading is increased, the same curve shows that the final pressure approaches the equilibrium value of 2100 lb/in.^{2}, and for 100 percent density of loading -- that is, $V_p/V_o = \infty$ -- equilibrium is reached at the very end of the process.

For lower values of the equilibrium pressure, say $\lambda_e = 0.0025$, or a pressure of 525 lb/in.^{2}, the approach to equilibrium tends to be more rapid, but it is still not completed. A similar situation is predicted if the initial pressure is higher than the equilibrium value.

These predictions are only partially confirmed by experiment. While the evidence is still not complete, it appears

that in most practical cases, the pressure builds up to the equilibrium value quite rapidly and stays there until the combustion is completed. By a "practical case" is meant one where the density of loading exceeds, say, 25 percent and the burning time exceeds, say, 0.03 sec. Under these conditions the equilibrium pressure is quickly reached even for very low starting pressures.

On the other hand, if the burning is very rapid, lasting, say, 0.01 sec, the equilibrium pressure is never reached. Also, if the density of loading is small, equilibrium is not attained even when the combustion lasts as long as 0.3 sec. The experimental curves appear quite similar to those in Fig. 3, even in respect to the effect of initial pressure.

In view of the only partial agreement between experiment and the simple theory used here and the lack of a criterion of applicability, we are forced to conclude that the theory cannot be relied on to predict the transient state. However, it does appear to predict equilibrium values with a reasonable degree of accuracy. Hence, for design work, a simple theory of this type is adequate for most purposes when tempered by a knowledge of experimental results.

It may be remarked here that, for the case of constant pressure, the burning time τ may be calculated from Eq.(2). This equation becomes, for this case,

$$\tau = D/(b + BP). \qquad (18a)$$

In design calculations, once the throat area A_t is known, the time τ may be also calculated from the equation

$$\tau = m_\tau/\dot{m}_d = m_\tau/\beta A_t P.$$

7. Exit velocities of gas in simple convergent nozzles and in expanding nozzles

On the assumption that the chamber pressure is never less than about 2 atmos and that the gas flow is steady, the usual engineering formulas for frictionless adiabatic flow give the gas velocity at the nozzle ~~throat~~ throat. as

$$w_t = \sqrt{2\gamma RT/(\gamma + 1)}, \qquad (19)$$

and the gas exit velocity from an expanding nozzle as

$$w_m = \sqrt{\frac{2\gamma RT}{\gamma - 1}\left[1 - \left(\frac{P_m}{P}\right)^{(\gamma-1)/\gamma}\right]}. \qquad (20)$$

If the nozzle is cut off at the throat, then Eq.(19) of course gives the exit velocity for a simple convergent nozzle. In Eq.(20), P is the chamber pressure and P_m is the pressure at the mouth of the nozzle. The latter is not necessarily the atmospheric pressure. If the nozzle under-expands, the pressure P_m is higher than atmospheric; this results in a decreased jet velocity which is partially compensated for by the thrust developed by the pressure in excess of atmospheric. If the nozzle over-expands the situation is quite complicated. An excellent treatment of these effects can be found in an article by F. J. Malina.[3]

3/ J. Franklin Institute 230, 433-451 (1940).

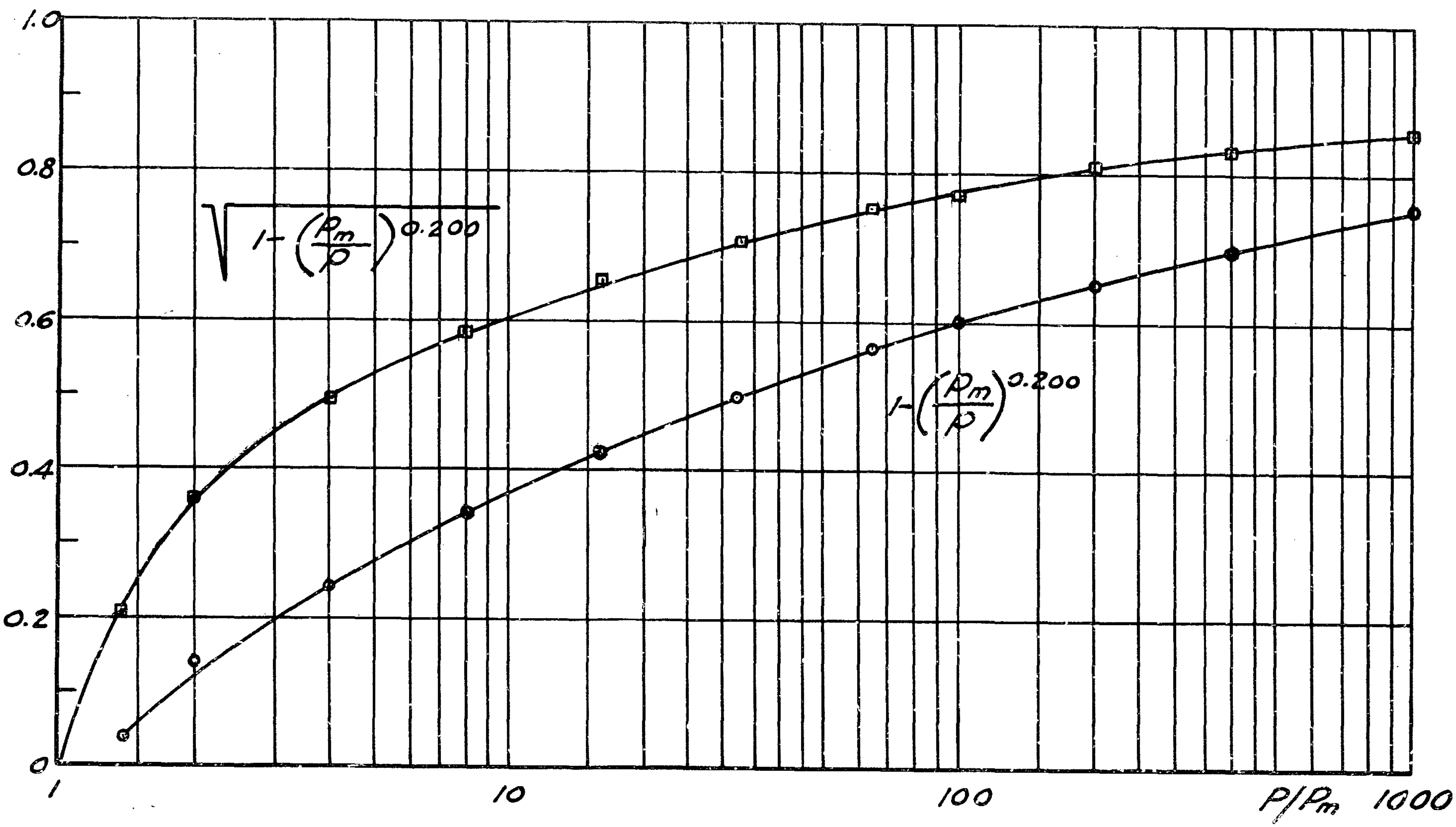

Fig. 1. Effect of finite pressure ratios on the gas exit velocity from an expanding nozzle.

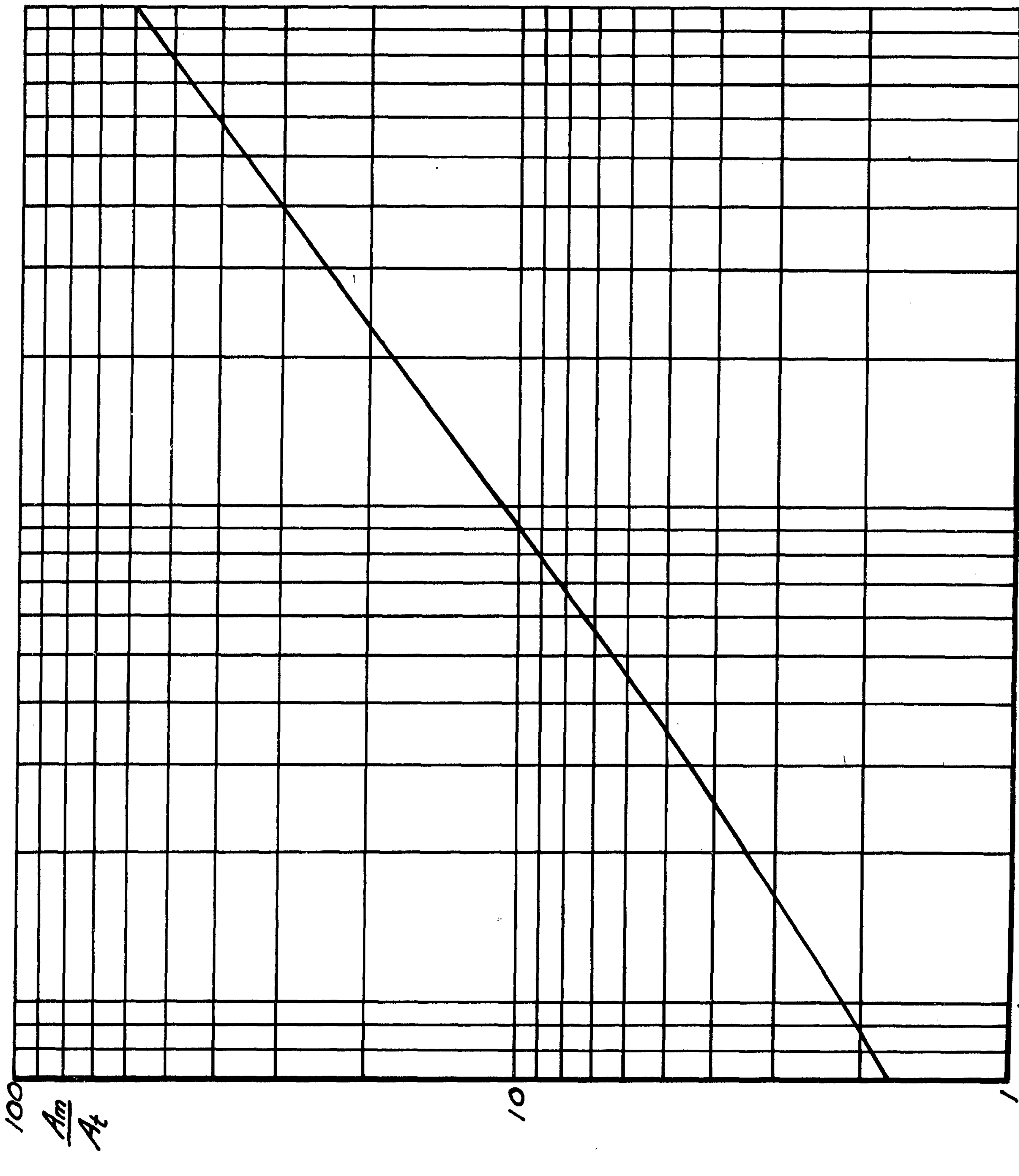

Fig. 2. Ratio of nozzle mouth area to nozzle throat area [Eq. (22a)].

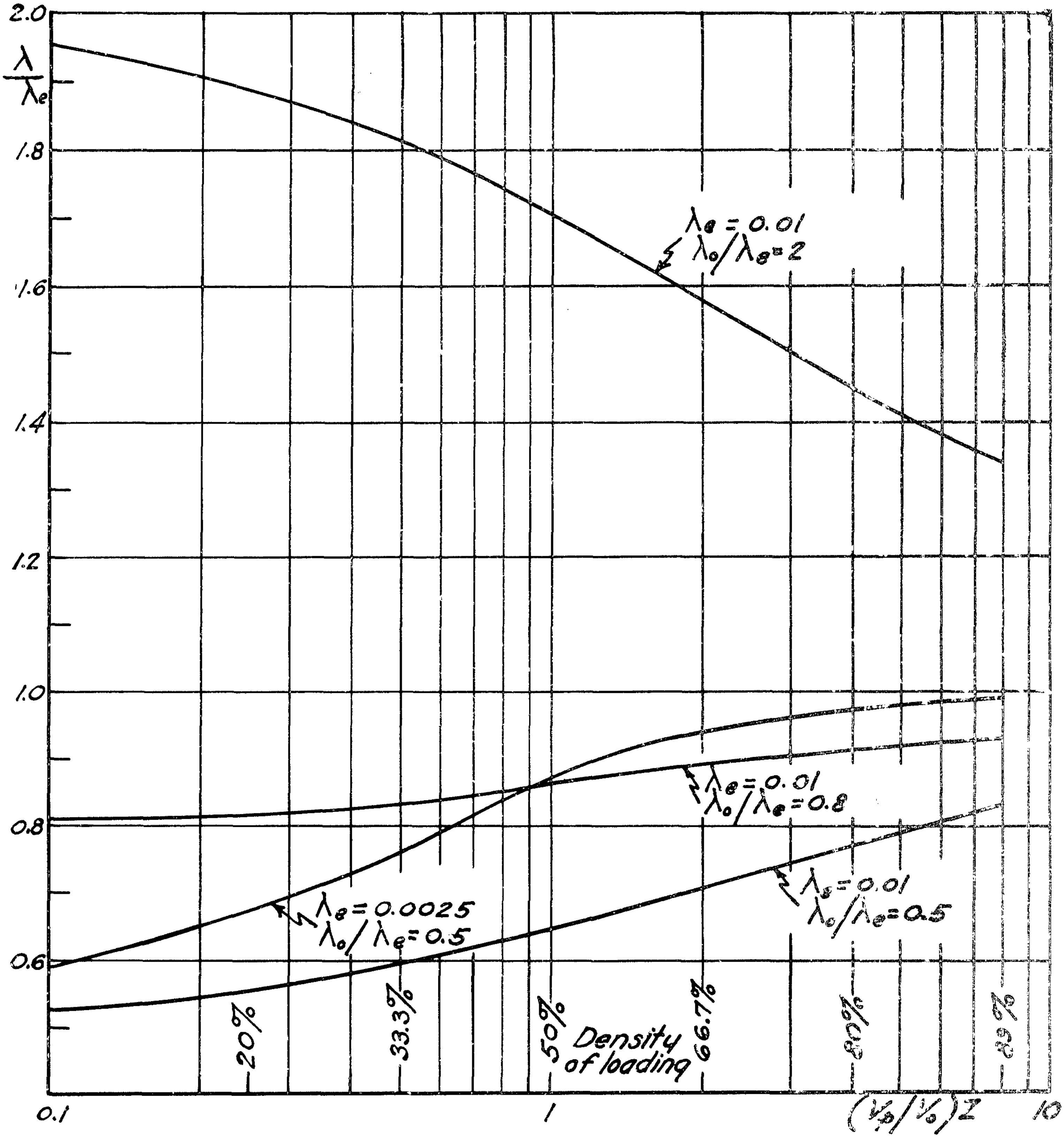

Fig. 3. λ/λ_e versus $(V_p/V_o)Z$ for several pairs of values of λ_e and λ_o/λ_e.

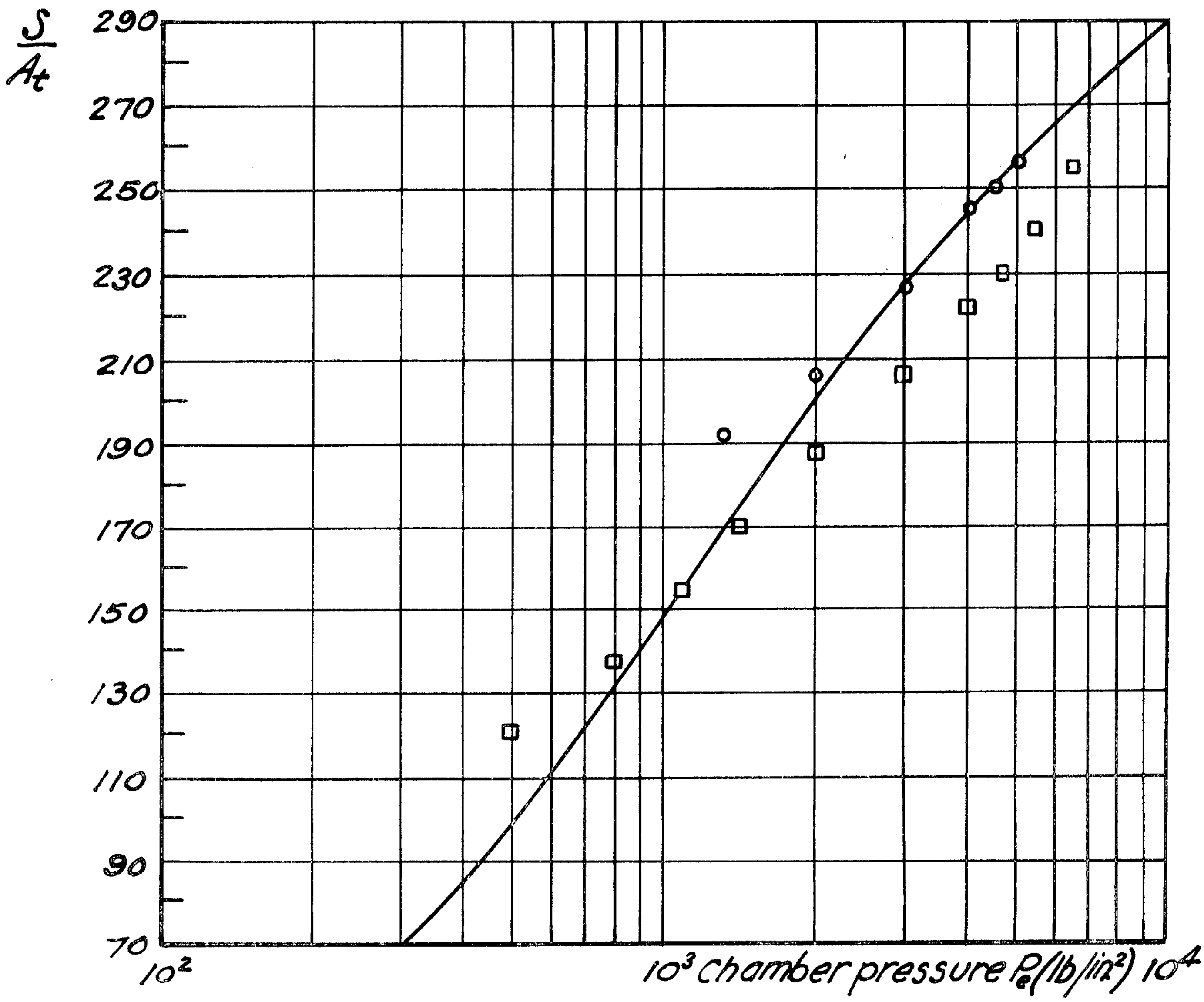

Fig. 4. Ratio of burning area to throat area versus equilibrium chamber pressure, for two different lots of powder (o, □).

For the propellant already referred to in Sec. 5, these gas velocities are

$$w_t = 3280 \text{ ft/sec} \tag{19a}$$

and

$$w_m = 9850 \sqrt{1 - \left(\frac{P_m}{P}\right)^{0.200}} \, . \tag{20a}$$

Note that, with this propellant, the highest velocity possible, (with infinite pressure ratio) is 9850 ft/sec. The effect of finite pressure ratios can be seen from Fig. 1.

It should be also remembered that the mass of gas discharged per second is given by Eq.(7) for both types of nozzles. Moreover, the pressure at the throat for both types is given by

$$P_t = P\left(\frac{2}{\gamma + 1}\right)^{1/(\gamma - 1)} , \tag{21}$$

or

$$P_t = 0.555 \ P \tag{21a}$$

for the propellant here considered.

8. Exit area

The standard formula for the nozzle mouth area A_m is[4/]

$$\frac{A_m}{A_t} = \left(\frac{P}{P_m}\right)^{1/\gamma} \sqrt{\frac{\frac{1}{2}(\gamma - 1)\left(\frac{2}{\gamma + 1}\right)^{(\gamma + 1)/(\gamma - 1)}}{1 - \left(\frac{P_m}{P}\right)^{(\gamma - 1)/\gamma}}} \, , \tag{22}$$

which, for the specific propellant here considered, becomes

$$\frac{A_m}{A_t} = \frac{0.208\left(\frac{P}{P_m}\right)^{0.800}}{\sqrt{1 - \left(\frac{P_m}{P}\right)^{0.200}}} \, . \tag{22a}$$

4/ For the theory of expanding nozzles see, for example, Martin, Textbook of mechanics, vol.V, "Thermodynamics", pp.230-236.

The plot of this function appears in Fig. 2. One should note that, in actual designs, it is frequently impracticable to use full expansion. That is, A_m is really fixed by the diameter of the projectile rather than by theoretical considerations. In such cases, Fig. 2 can be used to find the pressure drop when the areas are prescribed.

9. Comparison of thrusts obtained with a simple convergent nozzle and with an expanding nozzle for the case of constant pressure

Assume that the orifice area of the simple nozzle is the same as the throat area A_t of the expanding nozzle. The thrust F_t (lb) is the time rate of change of momentum (slug ft/sec²) plus the unbalanced force (lb) acting on the orifice area, or

$$F_t = \dot{m}_d w_t + A_t\,(P_t - P_a). \qquad (23)$$

For practical purposes the atmospheric pressure P_a may be neglected in comparison with the pressure P_t. If we then substitute the value of $\dot{m}_d$ from Eq.(7), w_t from Eq.(19) and P_t from Eq.(21), we get

$$F_t = 2A_t\,P\left(\frac{2}{\gamma+1}\right)^{1/(\gamma-1)} . \qquad (24)$$

For the propellant cited (Sec. 5),

$$F_t = 1.24\,A_t P, \qquad (24a)$$

The numerical factor in Eq.(24a) is called the thrust coefficient. It is a very convenient criterion for making comparisons in performance.

In the case of an expanding nozzle of correct ratio, that is, one which expands the gas to atmospheric pressure, the thrust is given by the first term of Eq.(23), except, of course, that the value of the exit velocity is now w_m, not w_t. Substituting the value of w_m from Eq.(11), we get

$$F_2 = A_t P\gamma \sqrt{\frac{2}{\gamma - 1}\left(\frac{2}{\gamma + 1}\right)^{(\gamma+1)/(\gamma-1)}} \cdot \sqrt{1 - \left(\frac{P_m}{P}\right)^{(\gamma-1)/\gamma}} \quad . \quad (25)$$

Numerically,

$$F_2 = 2.08\, A_t P \sqrt{1 - \left(\frac{P_m}{P}\right)^{0.200}} \quad . \qquad (25a)$$

By referring to Fig. 1 we find that, for a pressure of 200 atmos, the radical in Eq.(25a) is 0.81. In this particular case, the thrust coefficient would be 0.81 × 2.08, or 1.69. Comparing this value with that for the simple orifice [Eq.(24a)] we see that the thrust is improved in the ratio of 1.69 to 1.24, or 1.36. For lower chamber pressures, this improvement decreases, owing to the decreasing value of the radical.

For the case of an overexpanding nozzle, reference should be made to Malina's paper.[5/] The method to be used in the case of underexpansion is illustrated in Sec. 13.

10. Projectile velocity

To avoid the confusion that sometimes may arise, we will base the momentum calculations on first principles.

5/ See reference 3.

Consider a rocket of instantaneous mass $M_p(t)$ moving to the right and having, at time t, a velocity $\dot{x}$ relative to the ground. Take $\dot{x}$ as positive in the right-hand direction. An observer stationed on the rocket causes the release of a differential mass of gas dm_d with a velocity w_m, this velocity being measured relative to the rocket after the mass dm_d has been released. The observer takes the origin of his coordinate frame as being on the rocket. Relative to this frame he calls w_m positive when it is in the left-hand direction.

Since it is assumed that no external forces are acting, the law of conservation of momentum requires that the momentum before dm_d is released is equal to the algebraic sum of the momentums of the rocket and of dm_d after the latter is released. In other words, if the velocity of the rocket after the release of dm_d is $\dot{x} + d\dot{x}$, the law of conservation of momentum requires that

$$M_p(t)\dot{x} = [M_p(t)-dm_d]\,[\dot{x}+d\dot{x}] + [\dot{x}+d\dot{x} - w_m]dm_d,$$

or

$$0 = M_p(t)d\dot{x} - w_m dm_d. \qquad (26)$$

Since all the quantities in this equation are, in general, functions of the time,

$$\dot{x} = \int_o^t w_m \, dm_d/M_p(t),$$

or

$$\dot{x} = \int_o^t w_m \, \dot{m}_d dt/M_p(t), \qquad (27)$$

where

$$M_p(t) = M_o - \int_o^t \dot{m} \, dt, \qquad (28)$$

M_o being, of course, the initial mass of the loaded projectile.

Formally, at least, the problem of calculating the velocity is solved. Actually, in the general case, the function w_m is of such nature that any expression containing it is not likely to be integrable by any except numerical methods.

However, for the special case of a constant chamber pressure, the integration is quite simple because w_m and $\dot{m}_d$ are both constants. For this case, then,

$$\dot{x} = w_m \dot{m}_d \int_0^t \frac{dt}{M_o - \dot{m}_d t} = w_m \log \frac{M_o}{M_o - \dot{m}_d t} . \quad (29)$$

When the combustion ceases, at $t = \tau$, the chamber is still filled with gas, which then escapes at a constantly diminishing velocity. If the momentum of this residue is neglected, we may assume that

$$\dot{m}_d \tau \doteq m_\tau . \quad (30)$$

Making this substitution in Eq.(29), we obtain for $\dot{x}_1$, the maximum, or muzzle, velocity of the projectile,

$$\dot{x}_1 = w_m \log \frac{M_o}{M_o - m_\tau} . \quad (31)$$

The following relationships are sometimes useful:

(a) If M_∞ is the mass of the shell without propellant,

$$m_\tau = M_\infty \left(e^{\dot{x}_1/w_m} - 1\right) . \quad (32)$$

(b) If the ratio m_τ / M_o is sufficiently small,

$$\dot{x}_1 = w_m \left[\frac{m_\tau}{M_o} + \frac{1}{2} \left(\frac{m_\tau}{M_o} \right)^2 + \dots \right] . \quad (33)$$

For this condition, the acceleration of the projectile is approximately constant during the burning period.

11. Efficiency for the case of a constant chamber pressure

The expression for the efficiency is

$$\text{Efficiency} = \frac{\text{Kinetic energy of projectile at muzzle}}{\text{Heat energy of propellant}} ,$$

or

$$E = \tfrac{1}{2}[M_o - m_\tau]\dot{x}_1^{\,2}/J\,H\,m_\tau , \qquad (34)$$

where J is the mechanical equivalent of heat, which is equal to 778 ft lb/Btu, and H is the heat of combustion, expressed in British thermal units (Btu) per slug of propellant. But, according to Eq.(29)

$$\dot{x}_1^{\,2} = w_m^{\,2}\left[\log \frac{M_o}{M_o - m_\tau}\right]^2$$

and, according to Eq.(20),

$$w_m^{\,2} = \frac{2\,\gamma\,R\,T}{\gamma - 1}\left[1 - \left(\frac{P_m}{P}\right)^{(\gamma-1)/\gamma}\right];$$

hence

$$E = \frac{\gamma}{\gamma-1}\,\frac{R\,T}{J\,H}\left[1 - \left(\frac{P_m}{P}\right)^{(\gamma-1)/\gamma}\right]\left(\frac{M_o}{m_\tau} - 1\right)\left[\log \frac{M_o/m_\tau}{(M_o/m_\tau)-1}\right]^2 . \qquad (35)$$

But the gas constant expressed in heat units -- that is, R/J (Btu/slug ^oF) -- is merely the difference of the mean specific heats at constant pressure and at constant volume, provided the gas may be treated as ideal; in other words,

$$R/J = \bar{c}_P - \bar{c}_V. \qquad (36)$$

Moreover, since most of the combustion takes place under constant pressure, and in view of the simplification it introduces, the equation

$$H \doteq \bar{c}_p T \qquad (37)$$

is taken to represent the complete process. Combining Eqs.(36) and (37), we have

$$RT/JH = (\bar{c}_p - \bar{c}_v)/\bar{c}_p = (\gamma - 1)/\gamma. \qquad (38)$$

Let

$$M_o/m_\tau = \alpha'.$$

Then Eq.(35) becomes

$$E = \left[1 - \left(\frac{P_m}{P} \right)^{(\gamma - 1)/\gamma} \right] (\alpha' - 1) \left[\log \frac{\alpha'}{\alpha' - 1} \right]^2 . \qquad (39)$$

The form of the functions P_m/P and α' are shown in Figs. 1 and 5, respectively. The latter curve shows that the best value of α' is 1.25, which means that, of the projectile weight, 80 percent is propellant and 20 percent is shell. Assuming an infinite expansion ratio, we see that the greatest efficiency possible is 0.65.

As a more practical example, consider a 15-lb projectile, with 2.5 lb of propellant charge operating at 200 atmos pressure. Then $\alpha' = 6$, $\gamma = 1.25$, $1 - \left(\frac{P_m}{P} \right)^{(\gamma - 1)/\gamma} = 0.65$ and, from Eq.(39), $E = 0.65 \times 0.168 = 0.109$; that is, the efficiency is almost 11 percent.

It should be noted that Eq.(39), the expression for the efficiency, can be split into two factors. One depends only on the pressure ratio and is the thermal efficiency; it measures the fraction of the heat energy that goes into giving kinetic energy to the jet. The second factor, which depends only on α', the ratio of the initial total weight of the projectile to the weight of the propellant, is the propulsion efficiency; it measures the fraction of the jet energy that is converted into kinetic energy of the rocket itself.

Equation (39) is quite interesting in that it shows that when the weights of the propellant and of the projectile are fixed, nothing can really be done about increasing the efficiency.[6] The value of γ varies but slightly for various proportions of combustion products. The same thing is true for the factor $1 - (P_m/P)^{(\gamma - 1)/\gamma}$, for the pressure ratios in common use. Even H, the heat of combustion of the propellant, drops out of the efficiency expression. This is not to say that the heat of combustion H is unimportant. As will be shown in the next section, the muzzle velocity per unit mass of propellant varies as the square root of H. Hence, all other factors being equal, the substitution of propellants having larger heats of combustion will result in higher values of the muzzle

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6/ It is true the propulsion efficiency can be increased by giving the rocket an initial velocity. Whether the overall efficiency (including the energy required to impart the initial velocity) is also increased is rather problematical.

velocity. But, since $\dot{x}_1^2$ varies as $\underline{H}$, no higher efficiencies can be obtained.

12. <u>Muzzle velocity expressed in terms of the properties of the propellant</u>

Assume that $(m_\tau / M_o) \ll 1$ so that, in view of Eq. (32),

$$\dot{x}_1 = w_m \, m_\tau / M_o \, .$$

Hence the muzzle velocity per unit mass of propellant, namely $\dot{x}_1/m_\tau$, is given by

$$\dot{x}_1/m_\tau = w_m/M_o$$

or, in view of Eq. (20), by

$$\frac{\dot{x}_1}{m_\tau} = \frac{w_m}{M_o} = \frac{1}{M_o}\sqrt{\frac{2\gamma}{\gamma-1} RT \left[1 - \left(\frac{P_m}{P}\right)^{(\gamma-1)/\gamma}\right]}.$$

But

$$RT/JH = (\gamma-1)/\gamma ,$$

from Eq. (38), and therefore

$$\dot{x}_1 = \frac{m_\tau}{M_o}\sqrt{2JH\left[1 - \left(\frac{P_m}{P}\right)^{(\gamma-1)/\gamma}\right]} \, . \qquad (40)$$

Thus, the larger the heat of combustion of the propellant, the greater is the muzzle velocity per unit mass of propellant.

13. <u>Rocket design calculations</u>

The formulas developed in the preceding sections will now be applied to a specific design problem. Let it be required to calculate the mass of the propellant and the TNT load as a function of the chamber pressure and muzzle velocity for a

shell of diameter 3 in. and weight 15 lb, exclusive of the propellant.

(a) Calculations of muzzle velocities. -- The procedure followed can best be understood in connection with Table I. We first assume that the propellant is to be furnished with an outside diameter of 3/4 in. and an inside diameter of 3/16 in., so that the web thickness D is 9/32 in., or 0.0234 ft. A further assumption necessary is that the maximum outside diameter of the nozzle is the same as that of the shell, and that the wall thickness is 1/16 in. The nozzle mouth is then 2-7/8 in. in diameter and the exit area is 6.46 in.^{2}

We assume a fixed mass m_{τ} of propellant, in this case 3 lb(0.0930 slugs). The surface area S as found from Eq.(15a) is 369 in.^{2} Taking the chamber pressure as the independent variable [see line (1), Table I], we find the corresponding value of S/A_t from Fig. 4. Since S is 369 in.^{2}, the corresponding values of the throat area A_t can be found and listed [line (3), Table I].

The next step is to set down in line (4) the ratio of nozzle mouth area A_m to throat area A_t, it being known that A_m is 6.46 in.^{2} Then, from Fig. 2, we find the corresponding pressure expansion ratio, listed as $(P/P_m)'$ in line (5), Table I. A comparison with line (2) shows that the actual expansion ratio is appreciably smaller than the available pressure ratio. This is caused by the fact that the exit diameter of the nozzle is limited to the shell diameter. It therefore follows

TABLE I. Rocket design calculations.

m_τ = 3 lb* S = 369 in.^{2} A_m = 6.46 in.^{2} D = 0.281 in.

(1)	P(lb/in.^{2})	368	735	1470	2940	5880	11,760
(2)	P/P_m	25	50	100	200	400	800
(3)	A_t(in.^{2})	4.61	2.93	2.05	1.62	1.39	1.25
(4)	A_m/A_t	1.40	2.20	3.17	3.99	4.65	5.17
(5)	$(P/P_m)'$	5.0	10.4	18.2	26.0	32.0	37.0
(6)	Excess Force (lb)	380	361	427	636	1180	1960
(7)	w_m(ft/sec)	5210	6040	6530	6820	6950	7060
(8)	$\dot{m}_d$(lb/sec)	11.55	14.60	20.5	32.4	55.6	99.8
(9)	$m_d w_m$(lb)	1870	2740	4150	6860	12,000	21,900
(10)	F_2(lb)	2250	6820	4580	7500	13,200	23,900
(11)	w'_m(ft/sec)	6260	6820	7210	7450	7650	7720
(12)	$\dot{x}_1$(ft/sec)	1140	1240	1310	1355	1390	1405
(13)	Thrust Coeff.	1.32	1.44	1.52	1.57	1.61	1.63
(14)	m_c(lb)*	2.02	2.02	2.86	4.60	9.46	18.4
(15)	m_p(lb)*	0.13	0.19	0.26	0.37	0.53	0.74
(16)	Nozzle and Flange (lb)	0.27	0.53	0.66	0.73	0.76	0.78
(17)	ΣW(lb)	2.42	2.74	3.78	5.70	10.75	--
(18)	l_s(in.)	11.0	10.6	9.31	6.90	0.61	--
(19)	TNT (lb)	4.84	4.72	4.32	3.57	1.64	--

* These masses are expressed in pounds of mass; the values may be converted to slugs by dividing them by 32.2. The mass in pounds and the weight in pounds are practically equal numerically.

that the nozzle discharges the gases at pressures higher than atmospheric, and an unbalanced excess pressure will exist over the nozzle exit area. For example, for $P = 368$ lb/in.^{2} and a pressure expansion ratio of 5.0 (see the first column of data in Table I), the pressure P_m is 368/5.0, or 73.6 lb/in.^{2} Taking the atmospheric pressure as 14.7 lb/in.^{2}, the unbalanced pressure is 58.9 lb/in.^{2}, and this, acting over 6.46 in.^{2}, produces an excess force of 380 lb. Line (6), Table I, gives the excess force thus calculated for the other pressure ratios assumed.

The other component of thrust is that caused by the momentum of the escaping jet. The velocity w_m can be calculated from Eq.(20a), the value of the radical being read from the curve in Fig. 1. The pressure ratio to be used is that given in line (5). The rate of gas discharge $\dot{m}_d$ is found from Eqs.(7), (8) and (8a); that is,

$$\dot{m}_d = \beta A_t P,$$

the value of β being given in Sec. 5 as 11.97 slug/lb sec for the propellant used here. The rate of discharge of the gas is tabulated in line (8), and the jet reaction $\dot{m}_d w_m$ is listed in line (9).

The total thrust F_2 is obviously the sum of lines (6) and (9). From F_2 the effective jet velocity w_m' can be found; more specifically,

$$w_m'/w_m = \text{Column 10/column 9.}$$

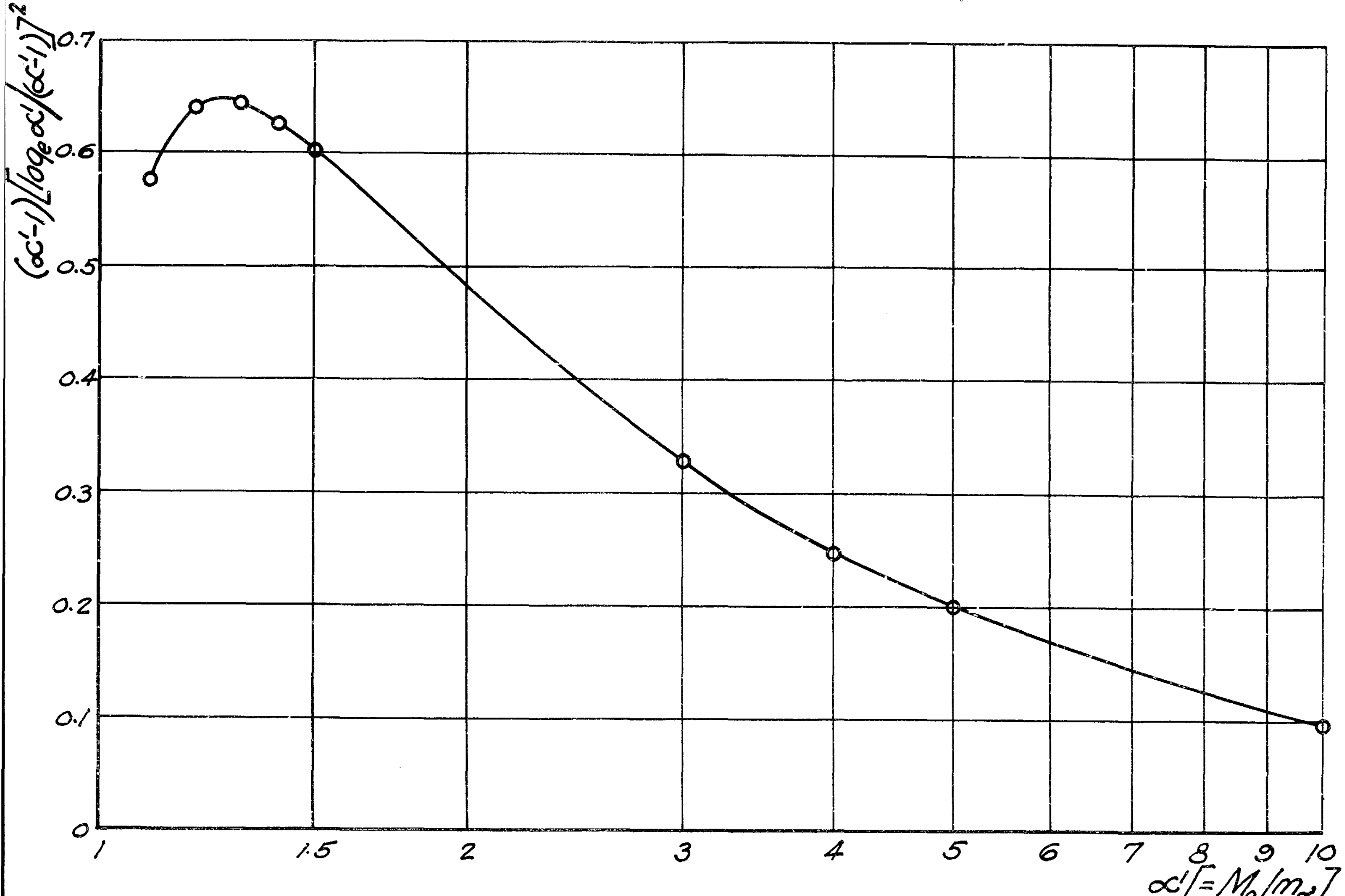

Fig. 5. Propulsion efficiency versus α'.

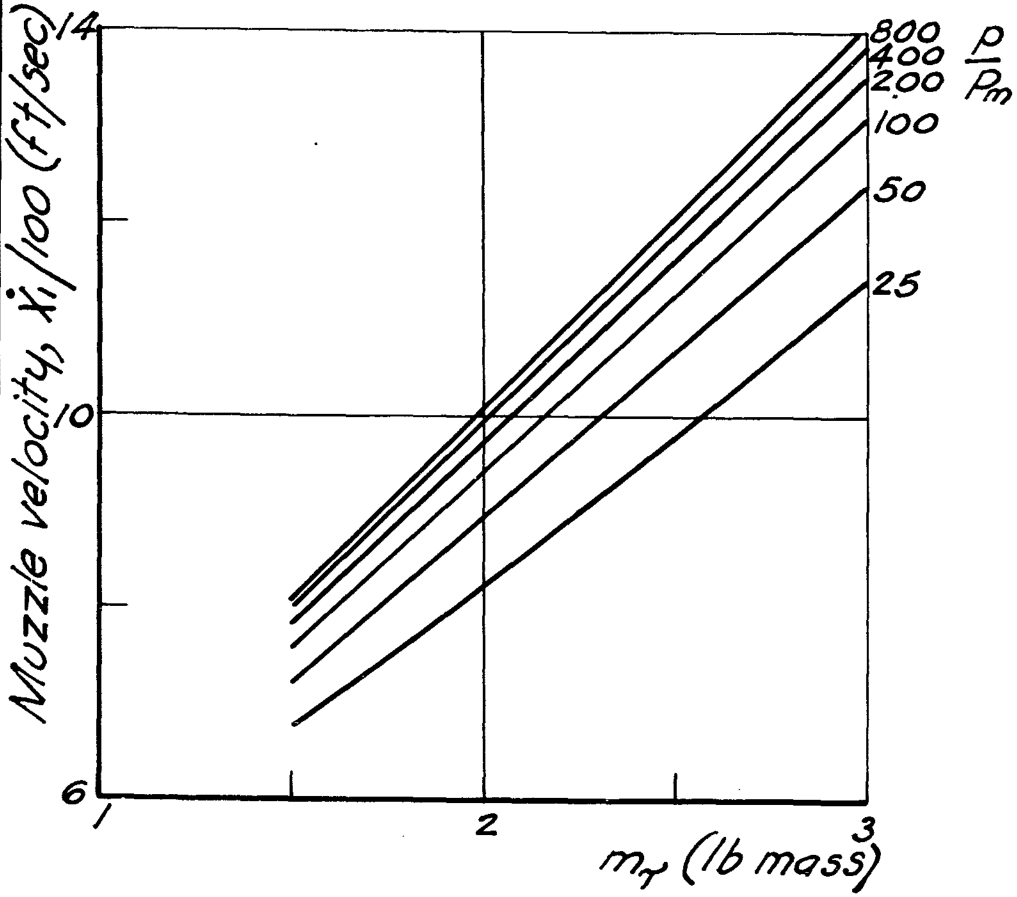

Fig. 6. Muzzle velocity as a function of propellant charge and pressure.

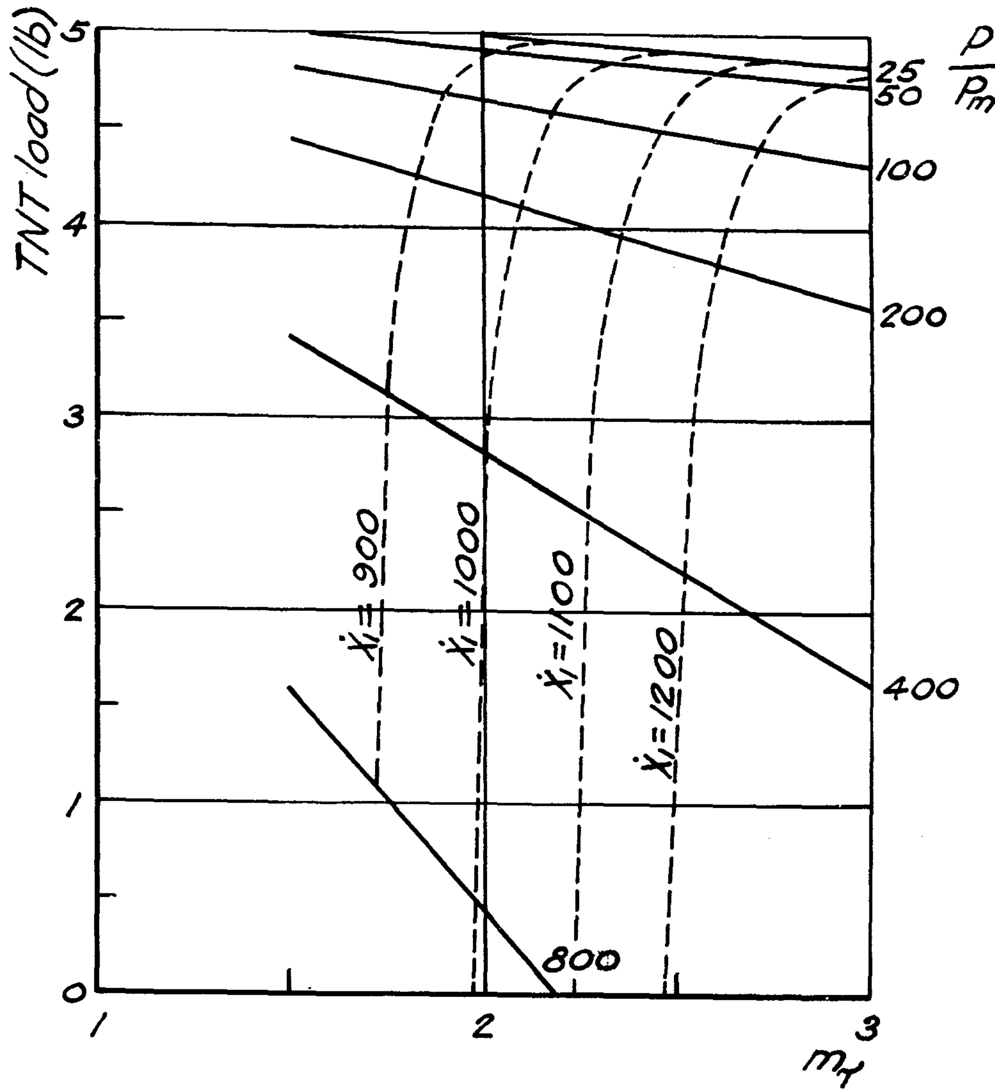

Fig. 7. TNT load versus propellant charge and pressure. The dotted lines are for constant muzzle velocity.

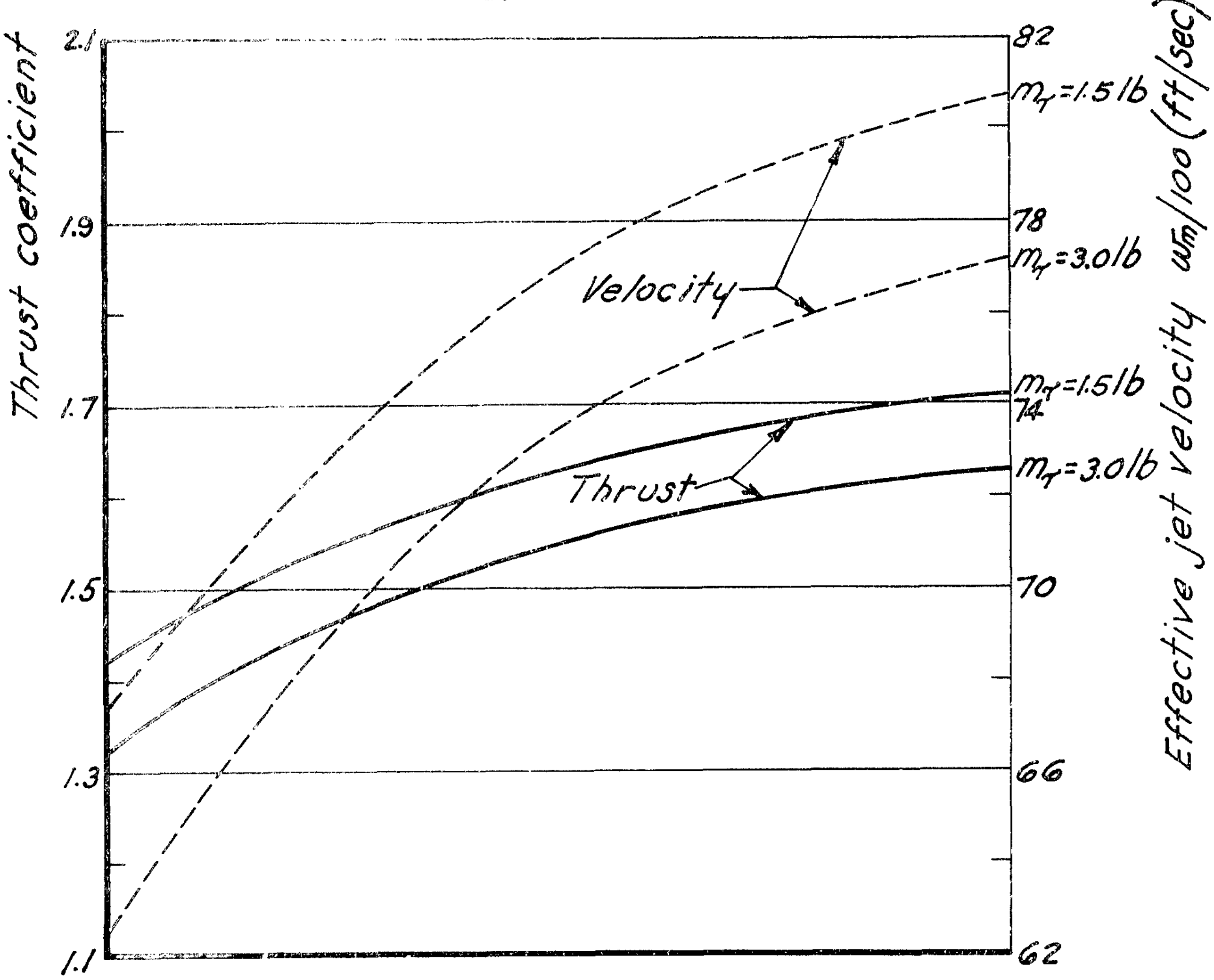

Fig. 8. Effective jet velocity and thrust coefficient versus pressure, for two different weights of propellant charge.

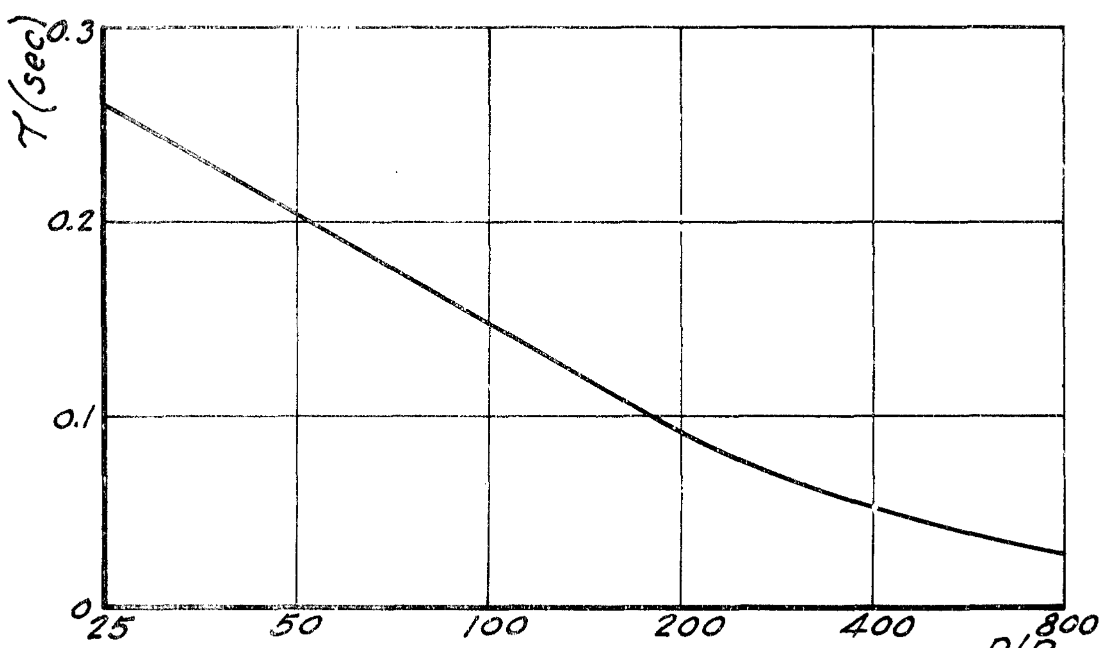

Fig. 9. Burning time versus pressure expansion ratio.

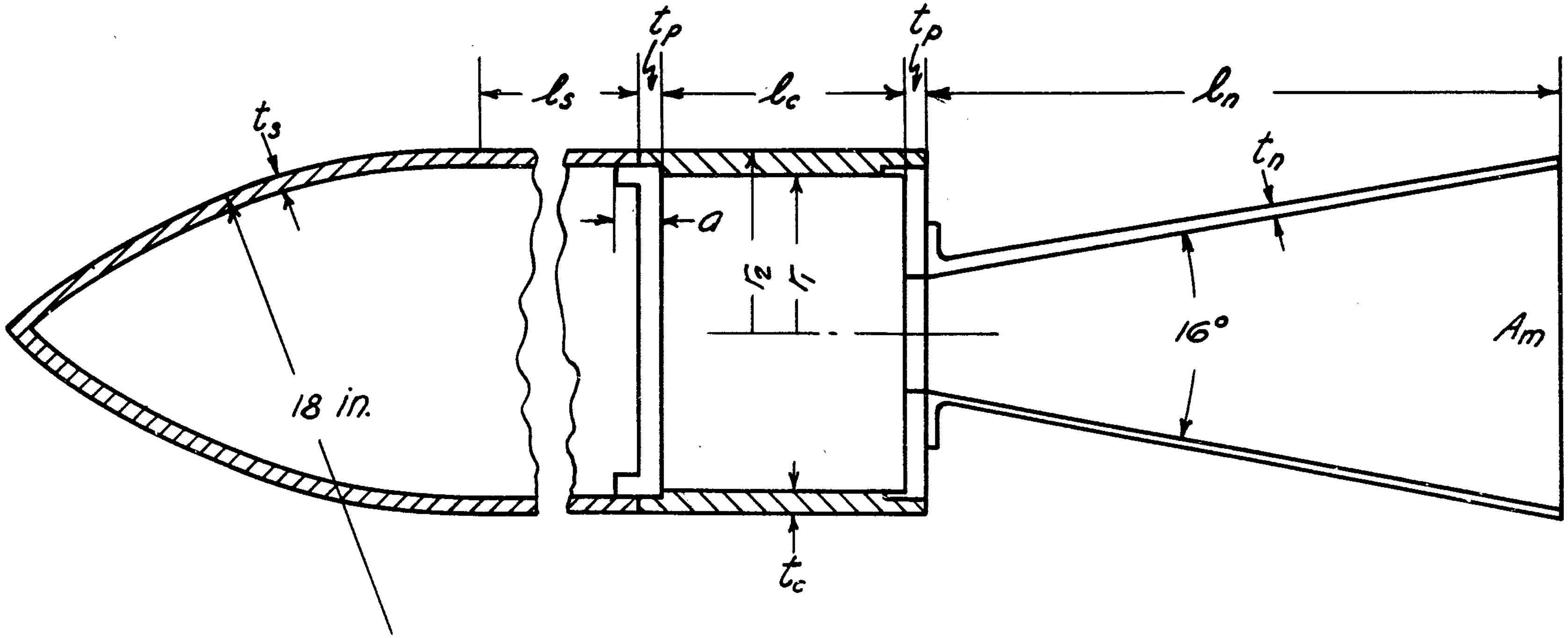

Fig. 10. Shell structure.

The muzzle velocity $\dot{x}_1$ can now be calculated from Eq.(31). The initial mass of the loaded projectile, M_o, is (15 + 3) lb, or 18 lb, w_m being taken as the effective velocity w_m' listed in line (11). The muzzle velocity is given in line (12). As a matter of interest, the thrust coefficient is given in line (13); it is calculated from Eq.(25a).

We now assume other values of the propellant mass and repeat the calculations. The results are shown in Figs. 6 and 8. Figure 6 is a plot of the muzzle velocity as a function of propellant mass (expressed in pounds) for the various assumed pressure ratios. Figure 8 represents plots of the effective jet velocity and the thrust coefficient, each as a function of pressure and two different weights of propellant charge. Figure 9 gives the burning time as a function of the pressure expansion ratio. Since we have assumed the same web thickness in all cases, this curve is the same for all designs.

(b) Calculation of weight of TNT charge. -- The shell structure is shown diagrammatically in Fig. 10. The shell head is threaded on a flanged plate which forms one end of the combustion chamber. The cylindrical portion of the combustion chamber is welded to this plate, and the end plate which carries the nozzle is threaded into the chamber body. The following data are assumed. The shell nose and body are made of 3/16-in. steel and the nozzle is made from steel of thickness 1/16 in. The chamber and plates will have thicknesses that are determined by the pressure. The safe allowable

stress is taken as 50,000 lb/in.^{2} in direct stress and 37,500 lb/in.^{2} in shearing stress. The threaded flanged plate will have a flange 1/4-in. long and 3/16-in. thick as a minimum, unless the plate thickness is larger, in which case the flange will be eliminated. The plate thickness will be calculated for direct and shearing stresses, and the thicker plate will be chosen. The two plates will be assumed to be of the same weight.

The radius of the shell ogive is taken as 6 calibers. The surface of the nose is 44 in.^{2}, and the volume is 25 in.^{3}

The weights per unit volume of the propellant and of the TNT will both be taken as 0.058 lb/in.^{3} The weight per unit volume of the steel is taken as 0.28 lb/in.^{3}.

Shell nose and body. The nose weighs

$$44 \times (3/16) \times 0.28 = 2.31 \text{ lb.}$$

The shell body weighs

$$\pi \times 3 \times (3/16) \times 0.28\, l_s = 0.494\, l_s \text{ lb},$$

where l_s is the length of the shell. The volume available for TNT is $25 + \frac{\pi}{4} \times (2\frac{5}{8})^2 l_s$, or $(25 + 5.31\, l_s)$ in.^{3} The weight of TNT is $0.058\,(25 + 5.31\, l_s)$, or $(1.45 + 0.308\, l_s)$ lb. The shell length l_s will be determined later.

Weight of combustion chamber. The wall thickness is computed from Clavarino's formula,[7/]

$$r_2/r_1 = [(3\sigma + P)/(3\sigma - 4P)]^{\frac{1}{2}}, \qquad (41)$$

7/ For a derivation of this formula see Martin, Textbook of mechanics, Vol. III, "Mechanics of materials", pp.176-180, particularly exercise 214.

where r_2 [= 1.5 in.] is the outer radius of the chamber, r_1 is the inner radius and σ is the stress (lb/in.^{2}). The least acceptable thickness is assumed to be 1/32 in.

The length of the chamber, l_c, may be calculated from the propellant weight. Assume as before that the outside and inside diameters of the propellant are 3/4 in. and 3/16 in., respectively, and that 6 sticks are used. We first check the density of loading. The net frontal area of the 6 sticks is 2.00 in.^{2} At a pressure of 800 atmos the area of the combustion chamber is 4.50 in.^{2} Hence the density of loading is 2.00/4.50 or 0.445, which is an acceptable value. It will be somewhat less than this for lower pressures.

The length l_c can then be found from the relation

$$m_\tau = \delta l_c A_p,$$

where A_p [= 2.00 in.^{2}] is the cross-sectional area of the propellant, δ [= 0.058 lb/in.^{3}] is the density of the propellant and m_τ is the mass of the propellant. The chamber masses m_c as calculated from the thickness $r_2 - r_1$ and the length l_c are given in line (14), Table I.

End plates. To be on the safe side, the stresses in bending were calculated on the assumption that the plates were simply supported at the edges. The maximum stress σ is at the center and is given by

$$\sigma = 1.25\, P(r_2/t_p)^2, \qquad (42)$$

where r_2 is the radius and t_p is the thickness of the plate. For all cases it turned out that the bending stress was greater

than the shearing stress and hence the plates were designed for bending rather than shear. The weights of the two plates are given in line (16), Table I.

Nozzle. The weight of the nozzle was calculated on the assumption that the wall thickness was 1/16 in. and the cone angle was 16°. Allowing 20 percent increase in mass due to the flange and fillets, one may express the weight in pounds quite closely by the formula

$$W_n = 0.15\ (A_m - A_t). \qquad (43)$$

The calculated values are given in line (16), Table I.

Weight of TNT. At this point we can total up all the weights except that for the shell body and the weight of TNT located therein. The sum of the chamber, end plate and nozzle weights is given in line (17), marked ΣW.

To get the total weight, we add to ΣW the weight of the shell nose and body, which is $(2.31 + 0.494\ l_s)$ lb, and the weight of TNT, which is 1.45 lb for the TNT in the nose and $0.308\ l_s$ lb for that in the shell body. The grand total of these components must be 15 lb; that is,

$$\Sigma W + (2.31+0.494\ l_s) + (1.45+0.308\ l_s) = 15 \text{ lb},$$

or

$$l_s = 14.05 - 1.25\ \Sigma W.$$

The values of l_s, the length of the shell body, are given in line (18). Knowing this, the corresponding total weights of TNT can be calculated and are given in line (19).

Discussion. -- The results of these calculations are summarized graphically in Fig. 7. These curves show the TNT load as a function of the propellant charge m_τ for various values of the pressure. The dotted lines are the loci for constant muzzle velocity. For example, for every point on the dotted curve marked $\dot{x}_1 = 1000$, the corresponding design has a muzzle velocity of 1000 ft/sec. These curves were obtained by projecting, from Fig. 6, the points of intersection of constant $\dot{x}_1$-lines with constant P/P_m-lines on to the corresponding pressure lines of Fig. 7.

It appears that the largest load of TNT can be carried when the pressure is somewhere between 50 and 25 atmos. The weight of TNT will then be between 4.75 and 5 lb. For pressure either above or below this value, the "payload" decreases. This is to be expected. For very low pressures, the thermal efficiency is small and the propellant weight must be increased to attain a given muzzle velocity. This means a longer combustion chamber. For very high pressures, the thermal efficiency increases very slowly but the weight of the chamber goes up rapidly, and again the payload will decrease.

14. Performance of geometrically similar shells

These results are more general and therefore more widely applicable than they may appear offhand. While the calculations were made for a shell of specific size, it is easy to show that the conclusions as to optimum pressures apply to all shells geometrically similar to one considered here.

If the diameter and length are multiplied by the same scale factor $\underline{s}$, the available volume and weight of TNT will vary as $\underline{s}^3$. For constancy of energy (during fragmentation) per unit mass of shell nose or body, the thickness of these parts must therefore be increased by the same scale factor. Hence the weight of the shell and its TNT load will vary as $\underline{s}^3$.

If all shells in the series are to have the same muzzle velocity $\dot{x}_1$, it is clear from Eq.(32) that the mass of the propellant charge must likewise vary as $\underline{s}^3$, assuming that the effective jet velocity is the same for all shells. Let us see what this requires. Assume tentatively that all shells operate at the same chamber pressure. Then, since the throat area A_t and nozzle exit area A_m both vary as $\underline{s}^2$, A_m/A_t is constant. Hence all nozzles have the same expansion ratio and, since the chamber pressure was assumed to be the same for all designs, it follows that the effective jet velocity will be the same for the entire series of shells. To sum up to this point, we have the result that, in a series of geometrically similar shells, all working at the same pressure, the muzzle velocity and jet velocity will remain constant provided that the propellant weight is increased in proportion to the weight of the shell.

The argument is not quite complete for, up to this point, we have ignored the weight variations of the combustion chamber, end plates and nozzles. From Eq.(41), r_2/r_1 is the same for all shells for equal pressures and equal allowable stresses.

Hence the weight of the combustion chamber, which is proportional to $l_c(r_2^2-r_1^2)$, must vary as $\underline{s}^3$. Similarly, an examination of Eq.(42), which gives the stress in the end plates, shows that the thickness t_p must vary as $\underline{s}$ for constant stress and hence the weight, being proportional to $r_2^2 t_p$ must vary as $\underline{s}^3$. The nozzle weight will likewise be seen to vary as $\underline{s}^3$ if we remember that Eq.(43) conceals, in its numerical coefficient, the length of the nozzle. This length will vary as $\underline{s}$ provided that the same taper is used in all designs. From the foregoing arguments, it is evident that if the optimum pressure is somewhere between 25 and 50 atmos for a 3-in. shell, the same range of pressures will be the optimum for all geometrically similar shells. Since the optimum condition is not critical, we can expect that small deviations in proportions from the ones assumed here will not materially affect the conclusion as to the best pressure range.

There is one more point to notice. Constancy of pressure requires that S/A_t be constant. Hence $\underline{S}$ varies as $\underline{s}^2$. But, since the propellant mass m_τ varies as $\underline{s}^3$, the web thickness $\underline{D}$ must vary as $\underline{s}$. Again, since $\underline{S}$ is proportional to the length of the propellant stick and the mean diameter of the stick, the latter must also vary as $\underline{s}$. From this it follows that the burning time $\underline{\tau}$ will be proportional to the scale factor but that the number of sticks of propellant is to be kept the same for all designs.

APPENDIX

List of Symbols Used

A_m	Area of nozzle mouth (ft^2)
A_p	Cross-sectional area of propellant ($in.^2$)
A_t	Area of nozzle throat (ft^2)
$\underline{b}$	Propellant burning constant (ft/sec)
$\underline{B}$	Propellant burning constant [(ft/sec)/(lb/ft^2)]
$\underline{C}$	Design parameter
$\bar{c}_P$	Mean specific heat at constant pressure (Btu/slug ^oF)
$\bar{c}_V$	Mean specific heat at constant volume (Btu/slug ^oF)
$\underline{D}$	Web thickness of propellant (ft)
E	Efficiency
F_t	Thrust for simple nozzle (lb)
F_2	Thrust for expanding nozzle (lb)
$\underline{g}$	Acceleration due to gravity (ft/sec^2)
$\underline{H}$	Heat of combustion of propellant (Btu/slug)
$\underline{J}$	Mechancial equivalent of heat [= 778 ft lb/Btu]
$\underline{k}$	Mass of gas discharged per unit time, per unit of pressure [(slug/sec)/(lb/ft^2)]
l_c	Length of chamber (ft)
l_s	Length of shell body (ft)
m_τ	Mass of propellant charge (slug)
$\underline{\dot{m}}$	Mass burned per unit time (slug/sec)
m_c	Mass of gas in chamber (slugs)
$\dot{m}_d$	Mass of gas discharged per unit time (slug/sec)

$M_p(t)$	Instantaneous mass of projectile (slug)
M_o	Initial mass of projectile, including propellant charge (slug)
M_∞	Initial mass of projectile, exclusive of propellant charge (slugs)
$\underline{P}$	Chamber pressure (lb/ft^2)
P_a	Atmospheric pressure (lb/ft^2)
P_b	Bomb pressure (lb/ft^2)
P_e	Equilibrium chamber pressure (lb/ft^2)
P_m	Pressure at nozzle mouth (lb/ft^2)
P_t	Pressure at nozzle throat (lb/ft^2)
$\underline{R}$	Gas constant for an ideal gas (ft lb/slug ^oF)
r_1	Inner radius of combustion chamber (in.)
r_2	Outer radius of combustion chamber (in.)
$\underline{S}$	Total burning surface (ft^2)
$\underline{t}$	Time (sec)
t_p	Thickness of end plates (in.)
$\underline{T}$	Temperature of gas (^oF absolute)
$\underline{v}$	Specific volume of gas (ft^3/slug)
$\underline{V}$	Instantaneous volume of gas in chamber (ft^3)
V_o	Initial, or clearance, volume (ft^3)
V_p	Initial volume of propellant (ft^3)
w_m	Gas exit velocity (ft/sec)
w'_m	Effective gas exit velocity (ft/sec)
w_t	Gas velocity at nozzle throat (ft/sec)
W_n	Weight of nozzle and flange (lb)
$\dot{x}$	Velocity of projectile at time $\underline{t}$
$\dot{x}_1$	Muzzle velocity of projectile (ft/sec)

Z	Fractional mass of propellant burned
α	Propellant constant
α'	M_o/m_τ
β	Propellant constant (slug/lb sec)
γ	Adiabatic exponent for propellant gases
δ	Density of propellant (slug/ft^3, lb/in.^{3})
λ	Ratio of chamber pressure to bomb pressure
λ_e	Equilibrium pressure ratio
λ_o	Initial pressure ratio
σ	Stress (lb/in.^{2})
τ	Burning time (sec)

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ABSTRACT:

A theory has been developed for internal ballistics of rockets driven by colloidal propellants with various relationships derived that are usable in preliminary design work. Calculations are summarized on graphs for mass of propellants and TNT load as function of chamber pressure and muzzle velocity for design of three-inch diameter fifteen-pound shell. Largest load of 4.75 to 5.0 pounds of TNT can be carried when operating pressure is from 25 to 50 atm. This applies also to geometrically similar shells

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